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which is the parametric form of the equations of the cycloid. The cycloid thus generated is the same as if described by a simple rolling of a circle of radius $\frac{1}{2}r$. From

$$\frac{dy}{dx}=\frac{\sin 2\varepsilon}{(1-\cos 2\varepsilon)}=\cot\varepsilon=\tan(\tfrac{1}{2}\pi-\varepsilon),$$

it follows that the diameter $2r$ of the circle is always tangent to the curve, so that we get the points of the curve as well as their corresponding tangents at the same time, which is rather an advantage. The construction of the cycloid is thus simplified. Besides, it is of interest that the definition of the cycloid is thus given in a different way from the one commonly employed.

DEPARTMENTS.

PROBLEMS FOR SOLUTION.

ALGEBRA.

305. Proposed by S. A. COREY, Hiteman, Iowa.

Prove or disprove, that $\sum\limits_{n=1}^{n=\infty}\frac{1}{(2n-1)^2+4}=\frac{\pi}{4}-\frac{\pi}{8}\left(\frac{\cosh\pi}{\sinh\pi}\right)$.

Solution by J. SCHEFFER, A. M., Kee Mar College, Hagerstown, Md.

Taking the logarithm of the well known formula

$$\cos\tfrac{1}{2}\pi x=\left(1-\frac{x^2}{1}\right)\left(1-\frac{x^2}{9}\right)\left(1-\frac{x^2}{25}\right)\dots$$

and differentiating, we have

$$\tfrac{1}{2}\pi\tan\tfrac{1}{2}\pi x=\frac{2x}{1-x^2}+\frac{2x}{9-x^2}+\frac{2x}{25-x^2}+\dots$$

Putting $x\sqrt{-1}$ in place of x, we have

$$\tfrac{1}{2}\pi\tan(\tfrac{1}{2}\pi x\sqrt{-1})=2x\sqrt{-1}\left[\frac{1}{1+x^2}+\frac{1}{9+x^2}+\frac{1}{25+x^2}+\dots\right]$$

Putting $x=2$, we have

$$\tfrac{1}{2}\pi\tan(\pi\sqrt{-1})=4\sqrt{-1}\left[\frac{1}{1+4}+\frac{1}{9+4}+\frac{1}{25+4}+\dots\right]$$

But, $\tan(\pi\sqrt{-1})=\dfrac{\tanh\pi}{\sqrt{-1}}$; $\therefore\dfrac{1}{1+4}+\dfrac{1}{9+4}+\dfrac{1}{25+4}+\dots=\tfrac{1}{8}\pi\tanh\pi$.

Also solved by G. B. M. Zerr, and V. M. Spunar.

306. Proposed by J. C. CORBIN, Pine Bluff, Ark.

Muir gives the following problem:

$$\text{Prove: }\begin{vmatrix}1 & a & a & a^2\\ 1 & b & b & b^2\\ 1 & c & c' & cc'\\ 1 & d & d' & dd'\end{vmatrix}=(a-b)\begin{vmatrix}1 & ab & a+b\\ 1 & cd' & c+d'\\ 1 & c'd & c'+d\end{vmatrix}$$

which, of course, can be solved by finding the terms of both determinants. Is there any method of changing from one form to the other which is direct?

Solution by J. A. CAPARO, C. E., Notre Dame University, Notre Dame, Ind.

Subtracting the first row from each one of the others:

$$\begin{vmatrix}1 & a & a & a^2\\ 0 & b-a & b-a & b^2-a^2\\ 0 & c-a & c'-a & cc'-a^2\\ 0 & d-a & d'-a & dd'-a^2\end{vmatrix}=(a-b)\begin{vmatrix}1 & 1 & b+a\\ d-a & d'-a & dd'-a^2\\ c-a & c'-a & cc'-a^2\end{vmatrix}=\triangle.$$

Multiply the first row by a and add it to the next two rows.

$$\triangle=(a-b)\begin{vmatrix}1 & 1 & b+a\\ d & d' & dd'+ba\\ c & c' & cc'+ba\end{vmatrix}=$$

$(a-b)[d'cc'+d'ba-c'dd'-c'ba-dcc'-dab+cdd'+abc+bdc'+adc'-bcd'-acd']$ which is the same as $[ab(c+d'-c'-d)+cd'(c'+d-a-b)-c'd(d'+c-a-b)]$ $\times(ab)$ or $\triangle=(a-b)[ab(c+d')-ab(c'+d)+cd'(c+d)-cd'(a+b)-c'd[d'+c)$ $+c'd(a+b)]$, which can be written

$$\triangle=(a-b)\left[\begin{vmatrix}ab & a+b\\ cd' & c+d'\end{vmatrix}-\begin{vmatrix}ab & a+b\\ c'd & c'+d\end{vmatrix}+\begin{vmatrix}cd' & c+d'\\ c'd & c'+d\end{vmatrix}\right]$$

which by Cor. 1, page 34, of Hanus' *Elements of Determinants,* is

$$\triangle=(a-b)\begin{vmatrix}1 & ab & a+b\\ 1 & cd' & c+d'\\ 1 & c'd & c'+d\end{vmatrix}.$$

Also solved by J. W. Clawson, and G. B. M. Zerr.